2

Chem. Eng. Comm. 1991, Vol. 103, pp. 131–149
Reprints available directly from the publisher.
Photocopying permitted by license only.

A NON-STEADY METHOD FOR DETERMINATION OF THERMAL DIFFUSIVITY AND THERMAL CONDUCTIVITY OF POORLY CONDUCTIVE SOLIDS

JOSÉ ANTONIO PIMENTEL, DAVID FAYÉS, AURA LÓPEZ-RAMOS and DOSINDA GONZÁLEZ-MENDIZABAL

Departamento de Termodinámica y Fenómenos de Transferencia,
Universidad Simón Bolivar,
Apartado Postal 89000, Caracas 1080, Venezuela.

(Received September 21, 1990; in final form February 11, 1991)

It is proposed in this work, to evaluate experimentally: thermal diffusivity, α, and thermal conductivity, k, of poorly conductive materials. The method selected is based on transient heat conduction in solid media and seeks to determine the conditions under which the power series solution characterizing the transient heat equation can be simplified and approximated, with a great degree of accuracy, by the first term of the series. It is then possible, by obtaining the temperature history (profile) of a single arbitrary point in the solid media, to compute the values of α and k.

The experimental procedure requires relatively low cost apparatus and measurement equipment easy to use. In addition, from a theoretical standpoint, the procedure is independent of the initial boundary condition (Temperature at $t = 0$) and of the temperature sensor location (length). Most important, a prior knowledge of density and heat capacity is not required for the determination of the thermal conductivity.

Three different solid geometries (Cylindrical, spherical and cubic) and five materials (PMMA, HMWPE, POLYAMIDE 66, POM and PTFE) were selected for the measurements. The results compare favorably with actual reported steady state values, within 5 per cent or less.

KEYWORDS Thermal conductivity Thermal difusivity Thermal properties Transient conduction Non-steady method Polymer properties

1. INTRODUCTION

In all heat transfer processes, it is necessary to predict the thermal behavior of the system under study, namely, the temperature distribution and the heat flux. These variables are a function of the nature of the solid, and consequently a function of certain physical properties.

Two of these properties, thermal conductivity (k) and thermal diffusivity (α), play a dominant role since α and k are to a certain degree related to the rate at which heat is transferred or internal energy stored.

There are several methods that permit the determination of α and k (Tye, 1969). Most of the methods are based in the integral form of Fourier's Law of heat conduction, when applied to steady state heat transfer processes. Fourier's

Law can be described by the following equation (Özisik, 1980):

$$q = k\,\frac{\Delta T}{\Delta x} \tag{1}$$

Equation (1) allows for a direct experimental determination of the value of k, under steady state conditions. This also requires to measure the temperature difference between two points in the direction of the heat flux. In addition, it is required that the heat flux be unidirectional (q a function of x only). Therefore, the z and y directions must be very large when compared to the x direction (x finite, $x \ll y$, $x \ll z$).

Two of these steady state methods traditionally employed for the determination of the thermal conductivity, based on one dimensional heat transfer, are described by ASTM C177 and VDE 0304 standards. Although their mathematical models are simple, they need very elaborated experimental procedures and require complicated and expensive equipments. Additionally, when the solids are poor heat conductors, the measurements have the disadvantage that they are very time consuming (until steady state conditions are achieved), and it is usually very difficult to maintain steady state conditions for long periods of time.

Another method is one that is based on unsteady state heat conduction, frequently employed in the last few years due to the remarkable progress in high precision electronic instrumentation (Uno and Hayakawa, 1980; Singh, 1982; Ansari *et al.*, 1984; Griffith, 1985). The mathematical description is based on the use of Fourier's Law in a pointwise fashion, together with the principle of conservation of energy. This description implies the presence of other properties besides the thermal conductivity, in order to properly describe the energy storage capacity of the solid such as the density, ρ, and heat capacity, C_p. The equation describing unsteady state heat conduction in a solid is as follows:

$$\frac{\partial T}{\partial t} = \alpha\,\nabla^2 T \tag{2}$$

where $\alpha = k/\rho\ C_p$. Eq. (2) assumes that the solid medium is isotropic with respect to heat conduction, and that the properties are constant over time.

The solution of Eq. (2) depends on the solid geometry, the initial temperature and the boundary conditions, i.e., on the manner in which the solid interchange energy with the surrounding fluid. These solutions are always infinite series where each of its terms consists of the product of a numerical coefficient, a function of position and time. In addition, the series have the following characteristic in that each term varies in a decreasing manner. This suggests that by properly selecting variables, the series may be truncated to fewer terms without much loss in accuracy. In certain cases, it can even be reduced to a single term, provided the first term represents 99% of the summation value.

The method to be developed in this work is based on using only the first term of the series solution of Eq. (2), with certain boundary conditions needed to determine the correct range of applicability. The magnitude of the first term of the series must remain very large compared to the other terms of the series for high values of the Fourier modulus (for high values of time). Thus, if one determines experimentally the temperature history at a point inside the solid, and chooses the appropriate value of the Fourier modulus for which only the first term of the series predominates, the logarithm of temperature is a linear function of time. It is possible then, after applying a least square to fit the data, to determine the value of the straight line parameters (slope and ordinate) with a pair of measurements of temperature vs time. One can hypothetically use these as two equations to obtain k and the unit surface conductance h. Although the ordinate is very sensitive to small changes in the position or initial conditions, in other words, in the location of the temperature sensor element, this method is suggested by Carslaw and Jaeger (1959).

In this study, a method is developed to determine the thermal diffusivity as well as the thermal conductivity based exclusively in the evaluation of the time coefficient in the exponential function. Several cases are presented: evaluation of α in a heat transfer medium where the Biot number tends to infinity, after h is measured in a known "medium" where a standard is used. After this experiment, one can calculate the conductivity of a different material with similar geometry, with a prior knowledge of ρ and C_p. Lastly, it is also possible to evaluate k by submerging the unknown solid in the same conductive medium, without a knowledge of ρ and C_p. In all cases the apparatus and experimental procedure are very simple.

2. MATHEMATICAL MODEL

The solutions to Eq. (2) for the following geometries, cylinder, sphere and cube, are all of the form of an infinite series:

$$\Theta = \sum_{n=1}^{\infty} A_n f_n(x) g_n(t) \tag{3}$$

where,

A_n: Fourier coefficient for the initial condition with respect to the eigenfunction $f_n(x)$ of the Sturm–Liouville problem.

$f_n(x)$: Eigenfunction of the Sturm–Liouville problem.

$g_n(t)$: Exponential function.

The numerical coefficient (A_n) is in general, a slowly decreasing function and depends on the geometry, the boundary and initial conditions. The functions of position $[f_n(x)]$ are bounded and are of the Bessel or trigonometric type. The time function $[g_n(t)]$ is exponential and the time coefficient is again solely a function of the boundary conditions and the geometry, but entirely independent of the initial conditions and the location of the sensor. This function of time includes as the Fourier's coefficient, the eigenvalue of the corresponding Sturm–Liouville system, which is the root of a transcendental equation. In Tables I, II and III, all coefficients and functions are tabulated, in dimensionless form, for the geometries under study.

One can then conclude from the latter, that for a given value of x, each term of Eq. (3) is smaller than the preceding term, and therefore for a specific Fourier

TABLE I

Solutions of the Eq. (2) for spherical geometry: Temperature initial condition constant and uniform $[\Theta_{(0,\sigma)} = 1]$ and two different boundary conditions

	Boundary condition	
	Surface constant temperature (Infinite Bi): $\Theta_{(Fo,1)} = 0$	Convective media (Finite Bi): $\left.\frac{\partial\Theta}{\partial\sigma}\right\|_{\sigma=1} = -Bi\,\Theta_{(Fo,1)}$
A_n	$\frac{2(-1)^{n+1}}{n\pi}$	$\frac{2Bi[\delta_n^2 + (Bi-1)^2]\sin\delta_n}{\delta_n^2[\delta_n^2 + Bi(Bi-1)]}$
$f_n(\sigma)$	$\frac{\sin(n\pi\sigma)}{\sigma}$	$\frac{\sin(\delta_n\sigma)}{\sigma}$
$g_n(Fo)$	$\exp(-n^2\pi^2 Fo)$	$\exp(-\delta_n^2 Fo)$
Transcendental Equation		$\delta_n \cos\delta_n + Bi - 1 = 0$

number, the series can be approximated by the first term as follows:

$$\Theta = \Gamma \exp(-\Omega t) \tag{4}$$

where,

Γ: Coefficient which is only a function of the initial condition and the selected thermocouple location

Ω: Coefficient function of geometry, boundary conditions and of the solid properties. In Table IV are listed the various expressions of the Ω coefficient for different geometries and boundary conditions.

Taking natural logarithm on both sides of Eq. (4) one obtains

$$\ln\Theta = \ln\Gamma - \Omega t \tag{5}$$

TABLE II

Solutions of the Eq. (2) for infinite cylinder: Temperature initial condition constant and uniform $[\Theta_{(0,\sigma)} = 1]$ and two different boundary conditions

	Boundary condition	
	Surface constant temperature (Infinite Bi): $\Theta_{(Fo,1)} = 0$	Convective media (Finite Bi): $\left.\frac{\partial\Theta}{\partial\sigma}\right\|_{\sigma=1} = -Bi\,\Theta_{(Fo,1)}$
A_n	$\frac{2}{\lambda_n J_1(\lambda_n)}$	$\frac{2Bi}{(Bi^2 + \lambda_n^2)J_0(\lambda_n)}$
$f_n(\sigma)$	$J_0(\lambda_n\sigma)$	$J_0(\lambda_n\sigma)$
$g_n(Fo)$	$\exp(-\lambda_n^2 Fo)$	$\exp(-\lambda_n^2 Fo)$
Transcendental Equation	$J_0(\lambda_n) = 0$	$\lambda_n J_1(\lambda_n) - Bi\,J_0(\lambda_n) = 0$

TABLE III

Solutions of the Eq. (2) for a cube. Temperature initial condition constant and uniform $[\Theta(0, \xi_x, \xi_y, \xi_z) = 1]$ and two different boundary conditions

	Boundary condition: Surface constant temperature (Infinite Bi): $\Theta(Fo, 1, \xi_y, \xi_z) = 0$; $\Theta(Fo, \xi_x, 1, \xi_z) = 0$; $\Theta(Fo, \xi_x, \xi_y, 1) = 0$	Boundary condition: Convective media (Finite Bi): $\left.\frac{\partial\Theta}{\partial\xi_x}\right\vert_{\xi_x=1} = -Bi\,\Theta(Fo, 1, \xi_y, \xi_z)$; $\left.\frac{\partial\Theta}{\partial\xi_y}\right\vert_{\xi_y=1} = -Bi\,\Theta(Fo, \xi_x, 1, \xi_z)$; $\left.\frac{\partial\Theta}{\partial\xi_z}\right\vert_{\xi_z=1} = -Bi\,\Theta(Fo, \xi_x, \xi_y, 1)$
A_n	$\frac{64(-1)^{n+m+i}}{\pi^3(2n+1)(2m+1)(2i+1)}$	$\frac{8Bi^3}{(Bi^2+\mu_n^2+Bi)(Bi^2+\mu_m^2+Bi)(Bi^2+\mu_i^2+Bi)\cos\mu_n\cos\mu_m\cos\mu_i}$
$f_n(\xi_x, \xi_y, \xi_z)$	$\cos\left[(2n+1)\frac{\pi}{2}\xi_x\right]\cos\left[(2m+1)\frac{\pi}{2}\xi_y\right]\cos\left[(2i+1)\frac{\pi}{2}\xi_z\right]$	$\cos(\mu_n\xi_x)\cos(\mu_m\xi_y)\cos(\mu_i\xi_z)$
$g_n(Fo)$	$\exp\left\{-[(2n+1)^2+(2m+1)^2+(2i+1)^2]\frac{\pi^2}{4}Fo\right\}$	$\exp[-(\mu_n^2+\mu_m^2+\mu_i^2)Fo]$
Transcendental Equation		$\mu_n \operatorname{tg}\mu_n = Bi$